

# Predictions for Triple Stars with and without a Pulsar in Star Clusters

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Accepted ; Received ; in original form

## ABSTRACT

Though about 80 pulsar binaries have been detected in globular clusters so far, no pulsar has been found in a triple system in which all three objects are of comparable mass. Here we present predictions for the abundance of such triple systems, and for the most likely characteristics of these systems. Our predictions are based on an extensive set of more than 500 direct simulations of star clusters with primordial binaries, and a number of additional runs containing primordial triples. Our simulations employ a number  $N_{tot}$  of equal mass stars from  $N_{tot} = 512$  to  $N_{tot} = 19661$  and a primordial binary fraction from 0 – 50%. In addition, we validate our results against simulations with  $N = 19661$  that include a mass spectrum with a turn-off mass at  $0.8 M_{\odot}$ , appropriate to describe the old stellar populations of galactic globular clusters. Based on our simulations, we expect that typical triple abundances in the core of a dense cluster are two orders of magnitude lower than the binary abundances, which in itself already suggests that we don't have to wait too long for the first comparable-mass triple with a pulsar to be detected.

**Key words:** globular clusters – pulsars, general – methods: N-body simulations – stellar dynamics.

## 1 INTRODUCTION

Numerical simulations of star clusters routinely produce dynamically formed triple stars, especially in the presence of primordial binaries (McMillan et al. 1990; Heggie & Aarseth 1992; McMillan & Hut 1994). Given that most globular clusters have a significant population of primordial binaries, we would expect some pulsar binaries in globular clusters to be actually part of a triple system. In such a case, most likely all stars have comparable masses, given the fact that encounters between binaries, and also between binaries and single stars, are likely to expel the lightest star, and hence subsequent encounters tend to concentrate the most massive stars in binaries and triples. Such a system we refer to as a *pulsar triple*, i.e. a triple system in which all components have comparable mass and one is a pulsar. (This phrase is preferable to the alternative *triple pulsar*, which would strongly suggest a triple system in which all

three components are pulsars.). The pulsar that is detected through its radio emission can then either be one of the inner two stars, or it can be the third star in orbit around the other two. The possible set of configurations is quite large: each of the other two stars can be a neutron star, a massive white dwarf, possibly a main sequence star, and perhaps even a black hole. So far, no pulsar triple with comparable masses has been discovered, among the more than 120 pulsars found in globular clusters.

Ideally one would like to explore the formation of pulsar triples using detailed N-body simulations which include direct integration of the orbits and stellar evolution (therefore using one particle per star). Unfortunately this approach is not computationally feasible today nor it will be in the near future. In fact, even if we consider a pulsar rich globular cluster such as Terzan 5, the population synthesis models of Ivanova et al. (2007), which include a simplified treatment of the dynamics of the system, predict a total of  $\approx 120$  neutron-star binaries (that is binaries with at least one neutron star) and thus about one neutron-star triple, considering that stable triples are about 1% of stable binaries (e.g.

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see Aarseth 2001; see also Sec. 6). This immediately highlights that, in order to gather enough statistics, one would need to run several tens of simulations of star clusters such as Terzan 5, which has a present mass of  $3.7 \cdot 10^5 M_\odot$  and a number of stars  $N > 4 \cdot 10^5$ . This is outside the capabilities of current hardware, including the GRAPE6: the largest direct N-body simulations carried out to date for at least a relaxation time are those by Baumgardt & Makino (2003) and have  $N=131072$  (see Hut & Trenti 2007). Thus one is forced to introduce some simplifications to address the formation of triples stars with a pulsar. As a first step we use equal-masses calculations as discussed in Sec. 3. These provide a reasonable approximation for the dynamics of old globular clusters, where the turn-off mass is below  $1M_\odot$  and the typical mass ratio between two stars in the cluster is around 2:1. Such an approach appears to be preferable to the use of simulations that start with very massive stars and up to several thousands particles, as is instead appropriate for the study of young open clusters (e.g. see de la Fuente Marcos 1996; Hurley et al. 2005). In addition, we validate our equal-masses results against simulations with a mass spectrum that has a turn-off mass at  $0.8M_\odot$ , consistent with the age of stellar populations in galactic globular clusters.

The aim of this paper is two-fold. First, we characterize the typical fraction of triples that are formed dynamically in a star cluster with a sizeable population of primordial binaries, comparing their number density with that measured at late times in simulations that start with primordial triples. Second, we take advantage of the fact that triple systems with one or more pulsars have most likely a dynamical and not primordial origin. We therefore apply our results to make an estimate of the likely time we have to wait until the first pulsar triple will be found as well as to predict its most likely characteristics.

The paper is organized as follows. In the next two sections we present an overview of observations and simulations, respectively. In Sec. 4 we present the initial conditions for the simulations presented in this paper. Sec. 5 briefly summarizes the global evolution of a star cluster with primordial binaries, extensively discussed in Heggie, Trenti & Hut (2006); Trenti, Heggie & Hut (2007); Trenti et al. (2007). In Sec. 6 we develop a simple model to describe the formation and destruction of triples. We use the model to interpret the observed triples fraction in our runs, attempting to characterize its  $N$ -dependence. In Sec. 7 we discuss in detail the properties of these dynamically formed triples. Sec. 8 describes the results of some additional runs where we have started with the presence of primordial triples. Sec. 9 presents the prospects for detection of a triple system with a pulsar and Sec 10 sums up.

## 2 OBSERVATIONS

There are currently  $\sim 130$  pulsars known in globular clusters, the majority of which are in binary systems (for a recent review, see Camilo & Rasio 2005)<sup>1</sup>. Since almost all of these systems are millisecond pulsars (MSPs) with extraordinary timing precisions (pulse arrival times are typically

measured to  $10\text{--}100\ \mu\text{s}$ ), if they were part of a triple system, the other two components would be easily detected, even if one of the components was low-mass or in a long-period orbit (Thorsett et al. 1999).

Since binary MSPs typically have orbital periods of several hours to several days, a pulsar triple system with the MSP in the internal orbit would likely be initially identified as a normal binary pulsar system. Timing observations would expose the third body in the external orbit after a time equal to a few percent of the external orbital period, which is likely to be less than a year.

Alternatively, a pulsar triple system could have the MSP in a relatively long-period (months to years) external orbit around a compact inner binary likely comprised of neutron stars, massive white dwarfs, or a combination of both, and possibly main-sequence stars. The MSP in such a system would be initially identified as a long-period binary or possibly even an isolated pulsar. Timing observations would relatively quickly reveal the MSP's external orbit, but unless strong orbital perturbations due to the inner binary are present, the internal orbit might never be detected and the MSP “companion” will be assumed to be a single massive star or compact object instead of a binary.

It is possible that stellar interactions could produce a triple system where two of the members are radio MSPs. The MSP in the external orbit would be relatively easy to detect due to the likely small orbital accelerations present over the short time intervals (i.e. hours) used for pulsar searches. For an MSP in the compact inner orbit, though, strong orbital accelerations would make the initial detection of the MSP very difficult, likely requiring specialized algorithms and extremely large amounts of computing (Ransom et al. 2003).

Finally one could ask what is the a-priori chance to find a triple system with two of the three bodies being pulsar. We expect that this probability is significantly smaller than that of finding a triple system with just one pulsar. In fact among the  $\sim 100$  binaries containing a pulsar, none is a double pulsar. This suggests that the probability for a triple with two pulsars is at least two orders of magnitude smaller than the probability of finding a single pulsar triple. The expected number of triples with two pulsars ( $N_{T-pp}$ ) can be estimated as:

$$N_{T-pp} = \eta \frac{N_p}{N_s} \frac{N_{B-ps}}{N_B} N_T, \quad (1)$$

where  $N_p$  is the number of pulsars,  $N_s$  is the number of single stars,  $N_B$  is the number of binaries,  $N_{B-ps}$  is the number of binaries with one pulsar,  $N_T$  is the total number of triples of the system and  $\eta$  is the fraction of pulsars not bound to any companion in globular clusters ( $\eta \approx 0.1$  from Freire 2005). If we assume for a globular cluster  $N_p \approx 10$ ,  $N_s \approx 10^5$ ,  $N_{B-ps} \approx 10$ ,  $N_B \approx 10^4$ ,  $N_T \approx 10^2$  we obtain  $N_{T-pp} \approx 10^{-6}$ . Therefore it is unlikely that any double-pulsar triple will be detected in globular clusters.

## 3 SIMULATIONS

The dense cores of globular clusters represent natural laboratories for studying exotic stellar populations, frequently living in multiple systems, such as blue stragglers, low mass X-ray binaries, cataclysmic variables, millisecond pulsars, stellar and, possibly, intermediate mass black holes. Most of

<sup>1</sup> See P. Freire's cluster pulsar catalog at <http://www2.naic.edu/~pfreire/GCpsr.html>

these objects form through the combined effects of stellar evolution and stellar dynamics, especially due to interaction and evolution of binary stars, that may constitute up to 50% of the core mass of a globular cluster (e.g., see Albrow et al. 2001; Bellazzini et al. 2002; Pulone et al. 2003). Observational evidence for triple stars is also starting to accumulate (e.g., see Raboud & Mermilliod 1998; Sterzik & Tokovinin 2002).

From the theoretical point of view, a detailed modeling of dense stellar systems is extremely challenging due to the huge dynamical range involved. Even neglecting stellar evolution and hydrodynamics and focusing on gravitational interactions only, the local orbital timescale of a binary star is several orders of magnitude smaller than the global relaxation timescale (hard binaries have an orbital period of a few hours, while the half mass relaxation time may be up to a few billion years).

Numerical simulations with primordial binaries have thus been performed either using approximate algorithms such as Fokker Planck or Monte Carlo methods (Gao et al. 1991; Giersz & Spurzem 2000; Fregeau et al. 2003), that have to rely on physical inputs on the interaction cross sections (but see Fregeau & Rasio 2007 and Giersz & Spurzem 2003 for Fokker Planck codes that treat binary interactions via direct N-Body integration), or using direct N-Body codes but employing, until recently, only a limited number of particles  $N \approx 10^3$  (McMillan et al. 1990; Heggie & Aarseth 1992; McMillan & Hut 1994; de la Fuente Marcos 1996). Studies of hierarchical systems in open clusters have been carried out by de la Fuente Marcos et al. (1997). A few direct numerical simulations with  $N \approx 10^4$  aimed at studying the formation and evolution of hierarchical systems have been recently performed by Aarseth & Mardling (2000); Aarseth (2001) and by Aarseth (2000, 2004).

To enhance the statistical base and remove some of the limitations of previous direct N-body investigations, we have started a project (Heggie, Trenti & Hut 2006; Trenti, Heggie & Hut 2007, hereafter HTH,THH respectively; see also Trenti et al. 2007) to study the evolution of star clusters with primordial binaries by means of several hundred direct numerical simulations employing up to  $N_{tot} = 19661$  particles. Our aim is to provide a milestone for comparison and validations of approximate methods and to investigate in detail the complex dynamical interactions of multiple stellar systems, that cannot so easily be characterized in terms of approximate treatments like Fokker Planck or Monte Carlo methods.

In this paper we take advantage of the extensive set of numerical simulations (more than 500) that we have performed (see HTH,THH) and we estimate the frequency and the orbital properties of dynamically formed triple stars. We discuss their dependence on the dynamical state of the star cluster (pre and post-collapse), on the initial binary ratio and on the number of particles  $N$ . We complement these simulations (i) with previously unpublished runs that include a mass spectrum appropriate to represent the collisional evolution of old stellar populations in globular clusters (that is with a turn-off mass below  $1M_{\odot}$ ) and (ii) with a few equal mass runs with a significant population of primordial triples. Although most of our simulations are limited to equal mass stars without stellar evolution, we briefly discuss in the concluding section the applicability of our results

to the prediction of the properties of globular cluster triple systems of comparable mass that include one, or more, millisecond pulsars.

## 4 INITIAL CONDITIONS

The general properties of the numerical simulations considered in this paper to study the properties of triple systems have been presented in detail in HTH and THH. To summarize, we use Aarseth's NBODY-6 code (Aarseth 2003) considering stars of equal mass and no stellar evolution. The initial distribution is either a Plummer model (HTH; in this case the system is considered isolated) or King models with concentration parameter  $W_0 = 3, 7, 11$  (THH; here the effects of a self-consistent galactic tidal field are also taken into account). The number of objects  $N$  used ranges from 256 to 16384; here  $N = N_s + N_b$ , where  $N_b$  and  $N_s$  are the initial number of binaries and single stars, respectively. The total number of stars is  $N_{tot} = N_s + 2N_b$ . In our standard runs (see HTH and THH), the primordial binary population has a fractional abundance ( $f = N_b/(N_s + N_b)$ ) from 0 to 50%, with an interal binding energy distribution flat in log scale in the range  $\approx 5 kT$ – $680 kT$ , where  $(3/2)kT$  is the mean kinetic energy per particle of the system (the binaries being replaced by their barycenter). In addition we have performed a few simulations with  $N = 8192$  and  $f = 20\%$  that start from a King  $W_0 = 7$  model (with a self consistent tidal field) and adopt an extended binding energy range (from  $5 kT$  to  $10^4 kT$ ) for primordial binaries. The aim of these runs is to specifically investigate the formation of triples originated from neutron star binaries with a degenerate companion.

In addition to these equal mass runs we also consider two tidally limited runs ( $W_0 = 7$  and  $W_0 = 11$  King models) with a mass spectrum. For these simulations, which have  $N = 16384$  and  $f = 10\%$ , the individual particle mass is drawn from an initial mass function appropriate to study the late evolutionary stages of star clusters, when stars are  $\approx 10$  Gyr old. To build the initial mass function for the run we first consider a Miller & Scalo (1979) mass function in the interval  $[0.2, 10]M_{\odot}$ , and then we proceed to take into account the effect of stellar evolution for massive stars by reducing the mass of particles above the turn-off (assumed at  $m_{toff} = 0.8 M_{\odot}$ ) accordingly to the prescription of Hurley et al. (2000).

All our results are presented using standard units (Heggie & Mathieu 1986) in which

$$G = M = -4E_T \equiv 1$$

where  $G$  is the gravitational constant,  $M$  the total mass, and  $E_T$  the total energy of the system of bound objects. In other words,  $E_T$  does not include the internal binding energy of the binaries, only the kinetic energy of their center-of-mass motion and the potential energy contribution where each binary is considered to be a point mass. The corresponding unit of time is  $t_d = GM^{5/2}/(-4E_T)^{3/2} \equiv 1$  (Heggie & Mathieu 1986). For the relaxation time, we use the following expression (Spitzer 1987)

$$t_{rh} = \frac{0.138 N r_h^{3/2}}{\ln(0.11N)}.$$

We recall that as our simulations consider gravitational in-

interactions only, our results, expressed in terms of the dimensionless units described above, can be applied to any physical choice for the total mass and scale radius of the system. The dependence of the triples' properties on the number of particles employed is discussed in Sec. 6 and provides a way to extrapolate the results of our simulations to a number of particles realistic for standard globular clusters.

For reference in physical units, a globular cluster (described by a Plummer model) with  $N = 3 \cdot 10^5$  stars, a total mass of  $M = 3 \cdot 10^5 M_\odot$  and a half-mass radius of  $4 pc$  has a half-mass relaxation time  $t_{rh} \approx 8.5 \cdot 10^8 yr$ ; in this cluster a binary, formed by equal mass stars each of mass  $1 M_\odot$ , with binding energy of  $1 kT$  has a semi-major axis of  $\approx 10 AU$  and an orbital period  $p_0 \sim 20$  yrs.

## 5 GLOBAL EVOLUTION

The evolution of the system in our runs is driven by the balance of two competing phenomena: the tendency of the core to contract (undergoing a gravothermal collapse) and the generation of energy due to binary-binary and binary-single interactions, that eventually halts the core contraction and fuels the half mass radius expansion. We can broadly identify two phases, extensively described in HTH and THH: an initial core radius adjustment and a quasi-steady binary burning phase.

The first “adjustment” transient is characterized by the evolution of the core toward a quasi-equilibrium configuration that will be maintained during the subsequent binary burning phase. In this phase we may have, depending on the initial conditions, a contraction (e.g., starting from a Plummer model, or from King models with  $W_0 \lesssim 7$ ) or an *expansion*, if the core is initially too small (Fregeau et al. 2003 and the discussion of the  $W_0 = 11$  runs in THH). For equal mass stars this transient lasts up to  $\approx 10 t_{rh}(0)$  and the expected core radius value at the end of this initial adjustment is well modeled in terms of general theoretical considerations (Vesperini & Chernoff 1994). Essentially the efficiency of the production of energy due to binary burning depends on the core density: if the core density is too low, the energy production is inefficient and the core shrinks; on the other hand a very high core density leads to an excessive energy generation, with a consequent core expansion.

After the “adjustment” transient, the details of the initial conditions are largely erased from the system and a self similar evolution sets in: the cluster expands by keeping the ratio of the core to half mass radius almost constant. The fuel for the expansion is provided by the hardening and destruction of the primordial binary population, that can sustain this phase for about  $100 t_{rh}(0)$  in isolated models (thus for a time much longer than the age of the universe for a typical globular cluster) if the initial binary fraction is above 10%.

As a result of four body interactions, basically happening in the core of the system only, where the interaction probability is highest, a (small) fraction of stable triple stars is formed. In the next Section we discuss the properties of these multiple systems.

## 6 TRIPLE FORMATION AND ABUNDANCES

Our runs start without primordial triples, but in a few relaxation times stable triples (identified as stable in NBODY6 using a semi-analytic approach based on the binary tides problem - see Aarseth & Mardling 2000 and Mardling & Aarseth 2001) are formed via dynamical interactions in binary-binary encounters following ejection of one star. We recall that stable triples cannot be formed due to three body encounters (see, for example, Heggie & Hut 2003).

After a first rapid rise of the number of triples that happens in the first few relaxation times, a quasi stationary regime sets in and the number of triples is approximately proportional to the number of binaries. This regime lasts for about 30 relaxation times, a time longer than the age of the universe for a typical globular cluster. Eventually, in isolated runs, as the evolution proceeds and the reservoir of primordial binaries is being depleted, the number of triples also drops. Fig 1 illustrates the evolution of the number of triples in an isolated run ( $N=16384$ , 10% primordial binaries, starting from a Plummer model) up to  $\approx 90 t_{rh}(0)$ . The drop in  $N_t/N$  for  $t > 30 t_{rh}(0)$  is clear, while the fraction of triples to binaries declines more slowly. Fig. 2 shows instead the number of triples for a run that includes a galactic tidal field, starting from a King  $W_0 = 7$  model with 20% primordial binaries and  $N=16384$ . The tidal field destroys the cluster in about  $30 t_{rh}(0)$ , thus even in the last stages of the evolution the binary fraction remains high (as single stars preferentially escape from the cluster) and so does the triple fraction. The asymptotic fraction of triple to single stars for our runs with equal-mass particles is given in Tab. 1. This has been obtained by averaging the triple to single stars ratio after the core contraction ( $t > t_{cc}$ ) and, in the case of tidal field runs, stopping when the number of stars in the system is reduced below 10% of its initial value.

The evolution of the system is very similar even when a mass spectrum is introduced in the simulations (see Fig. 3 and Tab. 2). Within the statistical uncertainties associated to the small number of triples present at any time in the system ( $N_t \lesssim 10$ ), the behavior of these runs is completely consistent with that observed in their equal-mass particles counterparts. These experiments with a mass spectrum confirm that equal mass runs are a very reasonable approximation as a first step for characterizing the properties of pulsar triples in systems where the turn-off mass is below  $1 M_\odot$ .

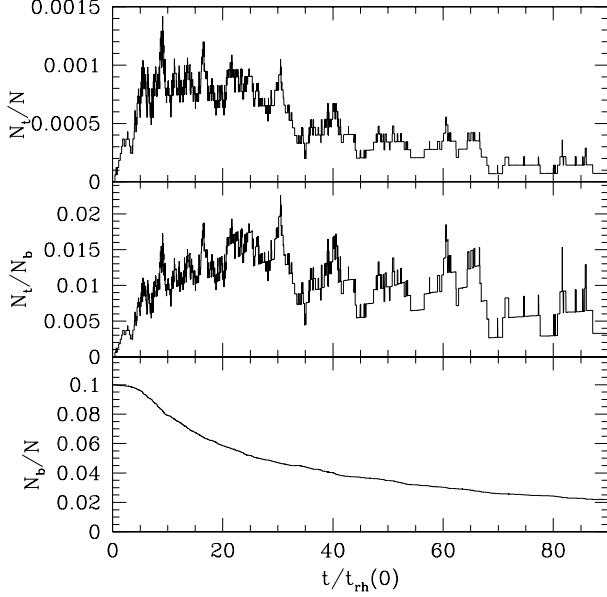
The evolution of the number of triples can be understood in terms of the balance between the formation ( $r_f$ ) and the destruction ( $r_d$ ) rate for triples. In first approximation we can write locally:

$$r_f = \alpha_f \cdot \rho_b^2, \quad (2)$$

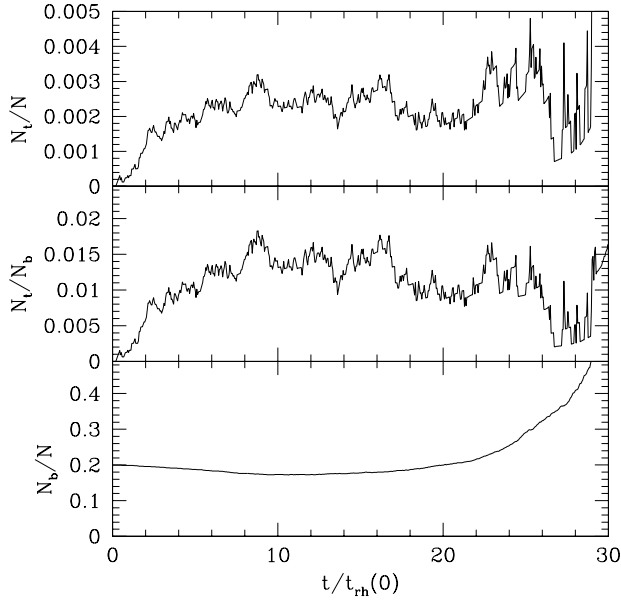
$$r_d = \rho_t (\alpha_{dB} \cdot \rho_b + \alpha_{dS} \cdot \rho_s), \quad (3)$$

where  $\rho_s, \rho_b, \rho_t$  are the density of singles, binaries and triples respectively, while the  $\alpha_i$  are numerical coefficients that depends on the cross section for the process considered.

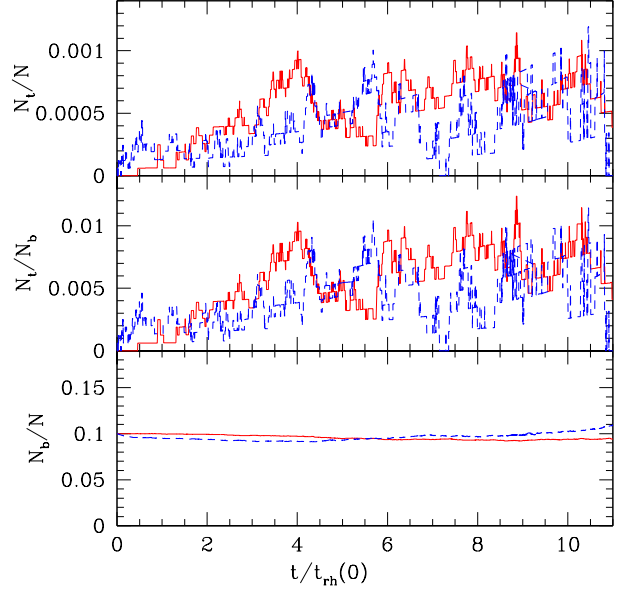
With the aid of Eqs. (2-3) we can explain the behavior of Fig. 2. During the initial core contraction phase the formation rate  $r_f$  is increasing as binaries accumulate in the core, while the destruction rate  $r_d$  is smaller due to the relative absence of triples, whose numbers therefore increase.



**Figure 1.** Evolution of the number of triples with respect to the number of particles  $N$  (upper panel), to the number of binaries (middle panel) and number of binaries to the number of particles (lower panel) for a simulation with  $N = 16384$  starting from a Plummer model (isolated, no tidal field) and 10% primordial binaries. Due to the absence of a tidal field the mass loss timescale is significantly longer than in the run presented in Fig. 2 and the drop in the number of triples at later stages of the evolution is apparent. See HTH for more details on the run.



**Figure 2.** Evolution of the number of triples like in Fig. 1 but for a simulation with  $N = 16384$  (with a tidal field) starting from a King  $W_0 = 7$  model and 20% primordial binaries. The time is in units of the initial half mass relaxation time  $t_{rh}(0)$ . After about  $30t_{rh}(0)$  the cluster is completely dissolved by the tidal field. See THH for more details on the run.



**Figure 3.** Evolution of the number of triples like in Fig. 1 but for two simulations with  $N = 16384$  (with a tidal field) starting from a King  $W_0 = 7$  model (red solid line) and  $W_0 = 11$  model (blue dashed line) with 10% primordial binaries. These simulations have particle masses drawn from a Miller & Scalo (1979) mass function evolved to have a turn-off mass at  $0.8M_{\odot}$ .

Around core collapse the formation rate reaches its peak and in the mean time an equilibrium regime sets in where the density of triples can be obtained by equating the formation and destruction rate ( $r_f = r_d$ ):

$$\rho_t = \frac{\alpha_f \cdot \rho_b^2}{\alpha_{dB} \cdot \rho_b + \alpha_{dS} \cdot \rho_s}. \quad (4)$$

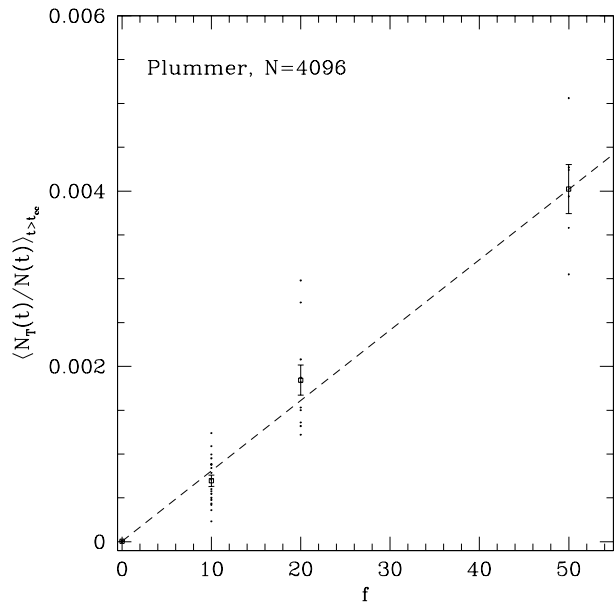
We can further assume, based on geometrical arguments, that the cross section for a binary-triple encounter is greater than that of a triple-single encounter. In addition we recall that after core collapse in simulations starting with  $f \lesssim 10\%$  the central binary density at least equals that of single stars (e.g. see Fig. 21 in HTH). This leads to a simplification of Eq. (4):

$$\rho_t = \frac{\alpha_f \cdot \rho_b^2}{\alpha_{dB} \cdot \rho_b} = \frac{\alpha_f}{\alpha_{dB}} \rho_b. \quad (5)$$

This theoretical prediction is nicely confirmed in Fig. 4, where the abundance of triples after core collapse for runs starting from  $N = 4096$  is shown as a function of the fraction of primordial binaries: here the structural properties of the system are nearly identical for  $f \gtrsim 10\%$  (see Figs. 17 & 18 in HTH) and there is a linear dependence of  $N_t$  on  $f$ .

Finally at later stages of the evolution of the system (i.e. well after the end of core-contraction phase),  $\rho_b$  in the core drops as binaries are being depleted, and the number of triples also decreases.

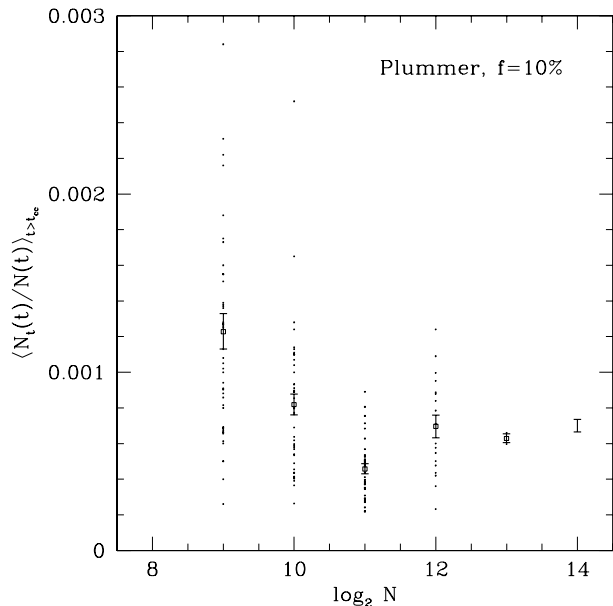
The fractional abundance of triples as a function of the number of particles used, for runs starting with  $f = 10\%$ , either from a Plummer or from a King  $W_0 = 7$  model, is depicted in Figs. 5-8. Especially for low  $N$  runs, before the core collapse the triple abundance is lower than after it.



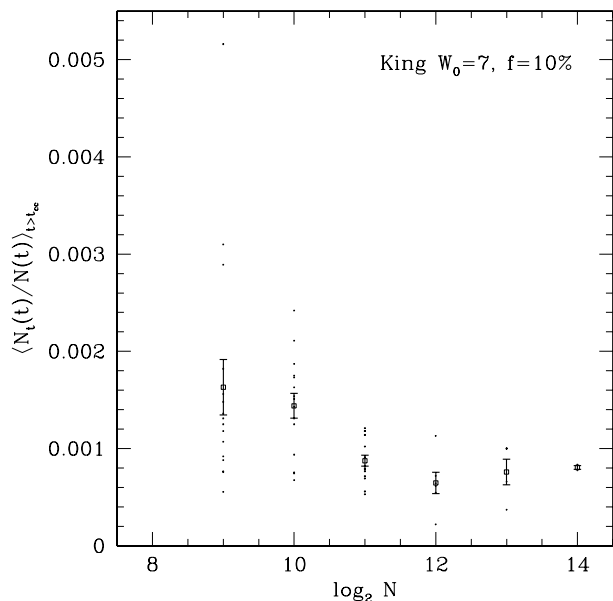
**Figure 4.** Fractional abundance of triples averaged after the core contraction for isolated runs starting from a Plummer model and different primordial binary ratios. Each dot represents a different run, while the squares, with a  $1\sigma$  error bar, are the average values of the fractional abundances of triples associated with a given primordial binary ratio  $f$ . The number of triples appears to be linearly proportional to the primordial binary fraction.

This is because the time for core collapse (in units of the relaxation time) increases with  $N$  (see HTH, Fig. 17), so that when  $N$  is low enough the system has not yet reached a balance between the formation and destruction rate of triples. In the runs starting from a King model we observe a marginally higher ratio of triples. This is due to the fact that the runs starting from a King model have a tidal field (see THH), so that the system steadily loses stars in its outskirts and the ratio of triples over total number of stars is enhanced in the post-collapse phase with respect to isolated runs starting from a Plummer model.

The  $N$  dependence of the number of triples at fixed initial binary ratio is more difficult to characterize than the dependence on the binary ratio at fixed  $N$ . In fact, while in the latter case the structural properties of the cluster are fixed, by varying  $N$  the core to half mass ratio is changing (see Vesperini & Chernoff 1994, and HTH, THH), so that the analysis in terms of Eqs. (2-3) is complicated by the necessity to integrate over the density profile and by the  $N$ -dependence of the  $\alpha$  coefficients that describe the efficiency of the different formation/destruction channels. Empirically we can also note the intrinsic large scatter in the measured triples abundances (see the dots representing individual runs in Figs. 5-8), so that it is hard to draw a firm conclusion. Tentatively, the  $N$  dependence is logarithmic at most, especially at lower  $N$  (in fact the core radius decreases approximately as  $1/\log(0.1 \cdot N)$ ). However for the last three points ( $N=4096, 8192, 16384$ ) the data are consistent with a constant value. In fact the decrease in the core radius is compensated by a higher central density and by an increased number of binaries over singles in the core (see



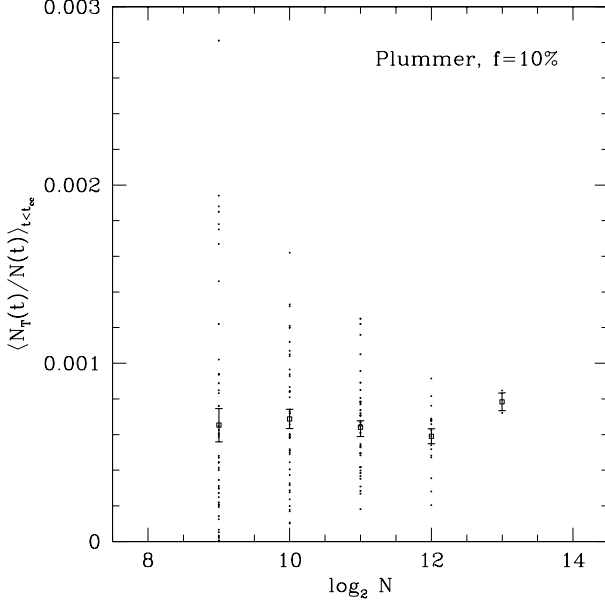
**Figure 5.** Fractional abundance of triples averaged after the core contraction for isolated runs starting from a Plummer model with 10% primordial binaries and different numbers of particles. Symbols are as in Fig. 4



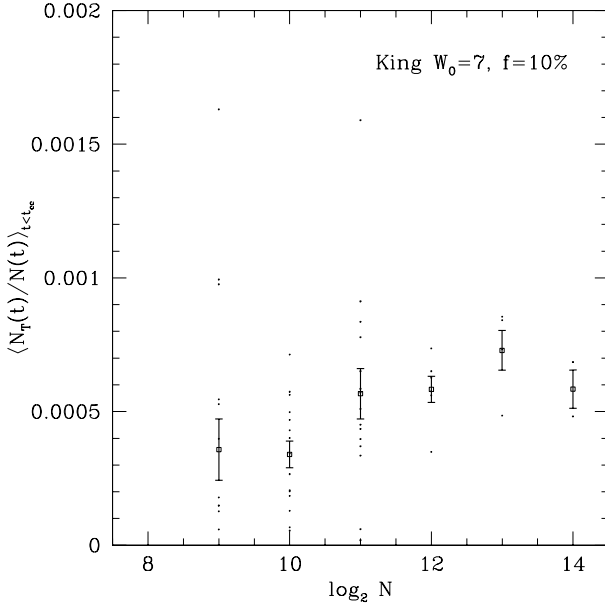
**Figure 6.** Like Fig. 5 but for simulations with a tidal field starting from a King  $W_0 = 7$  model.

Fig. 21 in HTH), so that the fractional abundance of triples should stay approximately constant.

The evolution proceeds in a qualitatively similar way in our runs initialized with an extended binary binding energy range (up to  $10^4 kT$ ). The only difference is that, all other conditions being equal, triples formation is enhanced by a factor between 1.5 and 2 when harder primordial binaries are present. In fact, binaries that have binding en-



**Figure 7.** Fractional abundance of triples averaged before the core contraction for isolated runs starting from a Plummer model with 10% primordial binary and different number of particles.



**Figure 8.** Like Fig. 7 but for simulations with a tidal field starting from a King  $W_0 = 7$  model.

ergies above  $\approx 700$   $kT$  are dynamically inert and behave essentially like singles during gravitational interactions with other stars. Therefore the encounter of one of these ultra-hard binaries with a regular (hard) binary can lead to an exchange encounter where one component of the regular binary is replaced by the ultra-hard pair, much like during a single-regular binary interaction. A more efficient production channel for stable triples is therefore available in these simulations. Therefore neutron star binaries with a degen-

**Table 1.** Asymptotic triple fraction in our equal mass runs. The first column reports the number of particles  $N$ , the second the initial density profile (Isolated Plummer model -  $Pl$  or tidally limited king model -  $W_0$ ), the binary fraction  $f$  is in the third column. The fourth and fifth entries are the average asymptotic triple fraction and its standard deviation, computed over  $N_{runs}$  realizations (last entry).

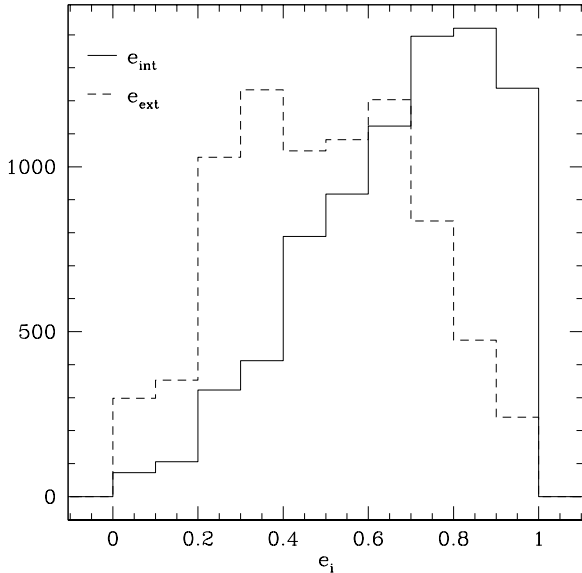
N	Model	f	$\langle N_t/N \rangle$	$\sigma_{N_t/N}$	$N_{runs}$
512	Pl	0	0.80E-04	0.11E-03	48
512	Pl	10	0.12E-02	0.68E-03	48
512	Pl	20	0.22E-02	0.81E-03	47
512	Pl	50	0.45E-02	0.14E-02	25
1024	Pl	0	0.35E-04	0.31E-04	49
1024	Pl	10	0.82E-03	0.39E-03	46
1024	Pl	20	0.19E-02	0.62E-03	49
1024	Pl	50	0.47E-02	0.13E-02	47
2048	Pl	0	0.13E-04	0.16E-04	46
2048	Pl	10	0.46E-03	0.16E-03	35
2048	Pl	20	0.17E-02	0.44E-03	39
2048	Pl	50	0.41E-02	0.93E-03	37
4096	Pl	0	0.50E-05	0.37E-05	7
4096	Pl	10	0.70E-03	0.27E-03	18
4096	Pl	20	0.18E-02	0.54E-03	11
4096	Pl	50	0.40E-02	0.62E-03	6
8192	Pl	0	0.18E-05	N/A	1
8192	Pl	10	0.63E-03	0.31E-04	2
8192	Pl	20	0.17E-02	N/A	1
16384	Pl	10	0.71E-03	N/A	1
512	$W_0 = 7$	10	0.16E-02	0.11E-02	16
1024	$W_0 = 7$	10	0.14E-02	0.50E-03	15
2048	$W_0 = 7$	10	0.87E-03	0.22E-03	15
4096	$W_0 = 7$	10	0.65E-03	0.27E-03	6
8192	$W_0 = 7$	10	0.76E-03	0.26E-03	4
16384	$W_0 = 7$	10	0.81E-03	0.28E-04	2
16384	$W_0 = 7$	20	0.15E-02	N/A	1
16384	$W_0 = 11$	10	0.54E-03	N/A	1
16384	$W_0 = 11$	20	0.22E-02	N/A	1

**Table 2.** Asymptotic triple abundance for tidally limited runs with a Miller & Scalo mass spectrum. Columns as in Tab. 1.

N	Model	f	$\langle N_t/N \rangle$	$\sigma_{N_t/N}$	$N_{runs}$
16384	$W_0 = 3$	10	0.98E-03	N/A	1
16384	$W_0 = 7$	10	0.68E-03	N/A	1
16384	$W_0 = 11$	10	0.61E-03	N/A	1

erate companion and with short orbital periods have an enhanced probability of ending up being in a triple system compared to main sequence binaries.

To summarize, in a typical globular cluster ( $N = 3 \cdot 10^5$ ) we expect an order of at least 100 triples if we conservatively assume a binary fraction of 10% (with a standard binding energy range, up to contact main sequence binaries) and  $N_t/N_b \approx 5 \cdot 10^{-3}$  (this accounts for the effect of a mass spectrum and tidal field in setting  $N_t/N_b$ ; see Fig. 3). The number approximately doubles by adopting an extended binding energy range that includes binding energies reached by neutron star binaries with a degenerate companion.



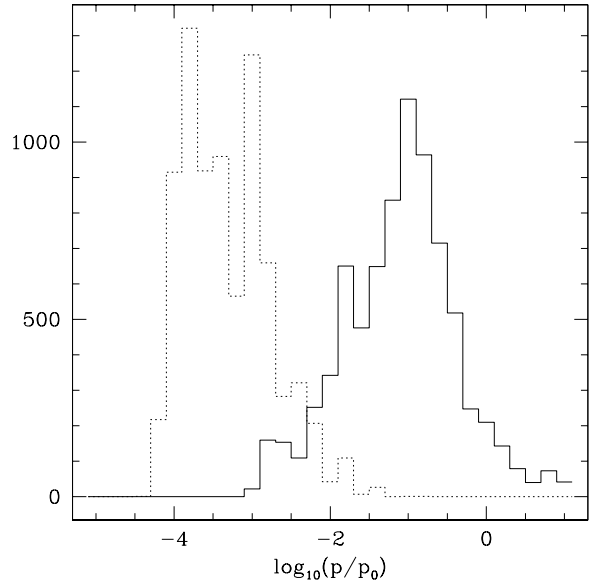
**Figure 9.** Internal ( $e_{int}$ ) and external ( $e_{ext}$ ) eccentricity distribution for triples measured in a series of simulations with a tidal field starting from King  $W_0 = 3, 7, 11$  models with 20% primordial binaries.

## 7 PROPERTIES OF NEWLY FORMED TRIPLES

While for the majority of the standard simulations in our sample we recorded only the number of stable triples, we have a small number of high resolution ( $N=16384$ ) simulations with complete information on the orbital properties of triples, saved every  $10 t_d$ . In addition, all the extended binding energy range simulations have recorded information of the orbital properties of multiple systems.

For a sample of 3 simulations with  $N=16384$  and  $f = 20\%$ , Fig. 9 shows the distribution of internal and external eccentricities. The inner eccentricity has a distribution peaked at high eccentricities which derives (i) from the input thermal distribution of the eccentricity for primordial binaries and (ii) from Kozai (1962) perturbations induced by the outer body, which lead to growth of the inner binary eccentricity (Aarseth 2001; Aarseth & Mardling 2000). Note that our simulations are limited to Newtonian dynamics, so we are missing relativistic corrections, which may alter the orbits of the inner binary (Aarseth 2007). The outer component of a stable triple is instead biased toward more circular orbits. This means that there is a selective disruption of triple systems with eccentric outer orbits.

For the same simulations the period distribution is reproduced in Fig. 10. Here we can note that the internal component has an orbital period between two and three orders of magnitude shorter than the outer component. The internal orbit corresponds on average to a binary with binding energy of a few hundreds of  $kT$  while the external orbit has a binding energy of a few  $kT$ . The distribution of orbital periods has a lower limit at  $\log(p/p_0) = -4.25$ , where  $p_0$  is the orbital period of a  $1kT$  binary. This lower limit corre-



**Figure 10.** Internal (dotted) and external (solid) period distribution - in units of the period for a  $1kT$  binary ( $p_0$ ) - for triples measured in a series of simulations with a tidal field starting from King  $W_0 = 3, 7, 11$  models with 20% primordial binaries and  $N = 16384$ .

sponds to a binding energy of  $\approx 700kT$ , i.e. a quasi-contact binary with  $1M_\odot$  main sequence components.

The dynamical interactions in the star cluster lead to a fast destruction of triples with external binding energy below  $1kT$ ; the destruction of the typical triples found in our simulations (with external binding energy of  $5kT$ ) happens on a timescale of about 10 half mass relaxation times.

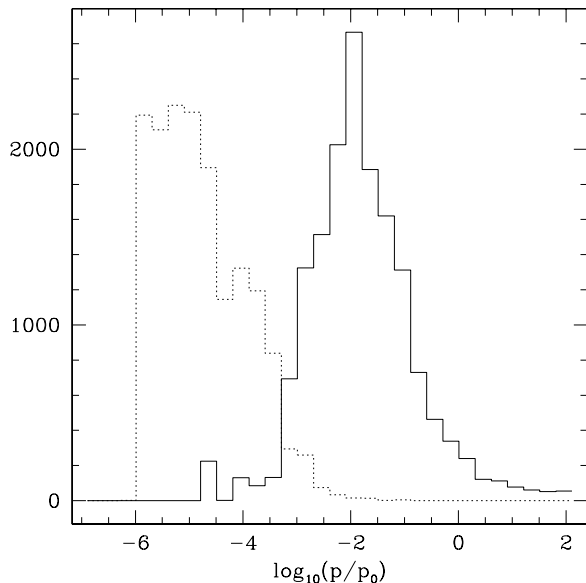
The eccentricity distribution in our extended  $kT$  range runs is similar to that plotted in Fig. 9, showing only a marginally less marked circularization of the outer orbit. The distribution of the orbital periods is instead different (see Fig. 11). Stable triples are composed preferentially of an inner ultra hard binary ( $E_b \gtrsim 500 kT$ ) and an external component that, on average, has a period only marginally shorter than in the standard runs.

The limited statistics in our simulations do not allow us to empirically constrain the  $N$ -dependence of the dynamical properties of triples. On the basis of theoretical modeling we do not expect significant variations with  $N$  for single mass star clusters, so that the orbital properties discussed here should be representative for simulations of globular clusters with a realistic number of particles. However one important caveat must be made as there is no guarantee that these properties are representative for the triple population formed in a realistic model that includes a mass spectrum and stellar evolution.

## 8 PRIMORDIAL TRIPLES

If a star cluster originates with a (small) population of primordial triples, how does the number of surviving primordial triples after several billions of years compare with the num-



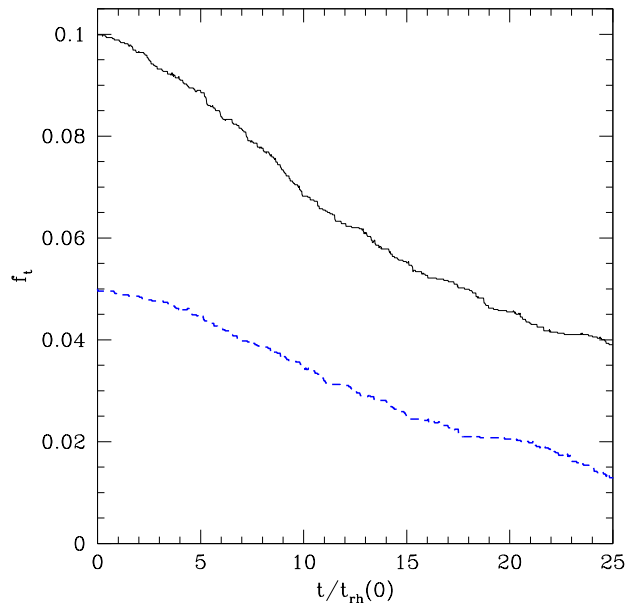


**Figure 11.** Internal (dotted) and external (solid) period distribution - like in Fig. 10 - for triples measured in a series of simulations with a tidal field starting from King  $W_0 = 7$  models with 20% primordial binaries,  $N = 8192$  and an extended initial binding energy range ( $[5 \text{ kT} : 10^4 \text{ kT}]$ ), representative for neutron star binaries with a degenerate companion.

ber of dynamically formed triples? To answer this question we have performed a few simulations with  $N = 4096$  starting from an isolated Plummer model and with a number of primordial triples ( $f_{trip} = 5\%$  and  $f_{trip} = 10\%$ , where  $f_{trip}$  is the initial number fraction of triples). These runs are, to the best of our knowledge, the first attempt to investigate the dynamical evolution of a star cluster with a number of particles and of primordial triples greater than the few hundreds particles used by van den Berk et al. (2006).

We started these runs by initializing the inner binary components with the same procedure adopted in Heggie, Trenti & Hut (2006), i.e. in an energy range  $[5 : 680] \text{ kT}$ . An external component has then been added using the triples initialization subroutine within NBODY6 (Aarseth 2003), with an energy in the range  $[0.1 : 100] \text{ kT}$  when the external energy is defined as  $2m/a_{ext}$  with  $m$  being the single particle mass and  $a_{ext}$  being the external semi-axis. This means that the softest external component has a semi-axis 100 times larger than the softest inner binary. The evolution of the number of triples is shown in Fig. 12: after a slow start (due to the initial large core, and therefore low density of triples in the core) primordial triples are steadily burned in the first  $15 t_{rh}(0)$  with a rate approximately proportional to the number of remaining triples (note that if multiplied by a factor 2 the curve for  $f_{trip} = 5\%$  reproduces closely the behavior of the curve for  $f_{trip} = 10\%$ ). This agrees well with the expectation based on Eq. 3.

At later times the destruction rate slows down slightly in Fig. 12 due to the expansion of the half mass radius, which increases the instantaneous relaxation time with respect to the initial one used in the figure. At about  $25 t_{rh}(0)$ , a time larger than the typical age of a galactic globular cluster, the



**Figure 12.** Evolution of the fraction of triples  $f_{trip}$  in two runs with  $N = 4096$  starting from an isolated Plummer model and  $f_{trip} = 5\%, 10\%$ . The destruction rate is qualitatively well described by Eq. 3.

number of primordial triples is higher than the number of dynamically formed triples from 10 – 20% primordial binaries runs, if the initial fraction of triples is higher than about one percent. If we add also primordial binaries in runs with primordial triples, the destruction rate of triples is slightly enhanced, as expected from Eq. 3 (we have verified this with a  $N=1024$  run with  $f_{trip} = 3\%$  and  $7\%$  primordial binaries).

## 9 DETECTION OF A TRIPLE SYSTEM WITH ONE PULSAR

As described in Sec. 4, for typical cluster parameters and  $1M_\odot$  stars, a  $1\text{kT}$  binary has a semi-major axis of  $\sim 10 \text{ AU}$ , and an orbital period of  $p_0 \sim 20 \text{ yrs}$ . Such an orbit is on the high-end of the external orbit distribution as shown in Figure 9. For an MSP in such a  $1\text{kT}$  orbit, the instantaneous orbitally induced period derivative would be  $\dot{P}_{orbit} \sim 10^{-14}$  (compared to a typical intrinsic  $\dot{P}_{PSR} \sim 10^{-20}$ , and 1-yr timing accuracies of  $\Delta\dot{P} \sim 10^{-22}$ ), and would be identified within a couple months of timing observations<sup>2</sup>. Shorter orbital periods would cause significantly larger period derivatives since  $\dot{P}_{orbit} \propto p^{-4/3}$ , where  $p$  is the orbital period. There is no evidence for such orbitally induced accelerations in any of the  $\sim 60$  globular cluster binary pulsars that have been or are currently being timed.

<sup>2</sup> This time is longer than what would be estimated by simply considering the six orders of magnitude difference in the accuracy of the 1-yr timing vs. the required accuracy for detection of a triple system. This is because there are many degeneracies (for example a positional offset) that can mask as a period derivative and that require a few months of observations before being excluded.

Over the past several years, intensive globular cluster pulsar searches have uncovered numerous systems that are almost certainly due to exchange interactions. These systems include at least 10 recycled pulsars in highly eccentric ( $e > 0.25$ ) orbits with periods between  $\sim 1$ –30 days (e.g. Freire et al. 2004; Ransom et al. 2005). Such systems are very similar to the external orbits found in the triples described in Figure 9. However, precise timing observations rule out the existence of internal orbits composed of at least one main-sequence star to a high degree of confidence. Surveys have also uncovered another class of exchange products, the 3 pulsar–“main-sequence” binaries NGC6397 A, Terzan 5 P, and Terzan 5 ad (D’Amico et al. 2001; Ransom et al. 2005; Hessels et al. 2006). These systems have circular orbits of duration 0.3–1.5 days and strange “bloated” companions that cause irregular eclipses. The low-eccentricity orbits were likely produced via tidal circularization. The erratic timing and irregular eclipses could be caused by an internal orbit, but the small orbital separations ( $2$ – $6 R_{\odot}$ ) and mass-function constraints indicate likely main-sequence dwarf companions.

The simulations we present suggest there should be approximately 100 times more binaries than triples in a typical globular cluster (see Figure 2). Given that we know of  $\sim 100$  globular cluster MSP binaries and have already one confirmed triple system, the strange MSP-WD-planet system in M4 (Thorsett et al. 1999), the current data are roughly consistent with the simulations. The next generation of radio telescopes, like the Square Kilometer Array, should uncover hundreds of new globular cluster MSPs, greatly increasing our chances of finding a pulsar triple system comprised of three stars of comparable mass.

## 10 DISCUSSION

In this paper we have presented a systematic investigation of the frequency of dynamically formed triples for an extensive set of direct simulations starting with a significant fraction of primordial binaries.

We have shown that on a timescale of a few relaxation times the abundance of triples reaches an equilibrium value that depends linearly on the primordial density of binaries and that is about two orders of magnitude smaller. Simulations including a tidal field present a relatively larger fraction of triples ( $N_t/N$ ) with respect to isolated runs with similar initial conditions. The presence of a mass spectrum tends instead to reduce the triple fraction.

Stable triple stars in our standard runs, representative for main sequence stars, are primarily formed due to four body encounters that lead to the escape of a single star. The formation mechanism influences the properties of these systems, which are primarily made of a hard inner binary, and which in a typical globular cluster would have an average orbital period of a few days, accompanied by an external star with a period on average between 100 and 1000 times longer. The inner component is also, on average, on a slightly more eccentric orbit than the outer member of the triple. In fact eccentric outer orbits are preferentially perturbed by gravitational encounters or break up the inner binary.

Neutron stars binaries with a degenerate companion can reach binding energy well above the  $700 kT$  limit of two main

sequence stars. To quantify the rate of formation of stable triples involving two degenerate stars we have carried out a series of simulations with binding energy distributed up to  $10^4 kT$ . In these runs we observe an enhanced triple abundance, as an ultra-hard binary made of two degenerate stars essentially behaves like a single body during gravitational encounters and can efficiently interact with a standard binary to form a triple via an exchange-like encounter. In a typical globular cluster, the inner orbital period may in this case be of the order of a few hours, while the outer component of the system is expected to be of the order of a few years, much like in the case of triples with main sequence stars.

Triple stars with at least one pulsar component are expected to have a dynamical origin, as it is highly unlikely that a pre-existing triple system can avoid being disrupted by the Supernova explosion that forms the pulsar. Therefore we expect that the properties of pulsar triples are in agreement with those derived by runs starting with primordial binaries only, where all the triples have been formed through stellar encounters (though a single pulsar could become a member of a primordial triple via an exchange process). To date,  $\sim 100$  pulsar binaries are known in the globular cluster system (Camilo & Rasio 2005). Based on our idealized simulations, we expect that the discovery of a pulsar in a triple system of comparable masses is likely to happen soon. However, we note that the majority of the known pulsar binaries have companions of mass  $\sim 10\%$  of the pulsar mass rather than  $\sim 1 M_{\odot}$ . How this difference affects our conclusions is currently unknown and will demand more detailed studies with a spectrum of stellar masses beyond that used in this study.

Higher order hierarchical systems, such as quadruplets and quintuplets are also occasionally observed during our runs, and their average number density is about two orders of magnitude smaller than that of triples. This makes a more detailed characterization of their properties extremely challenging, at least for simulations with only a limited number of particles like ours. A survey aimed at quantifying the observed fraction of multiple stars in globular clusters would allow us to understand if the observed frequency of triples and higher order systems is consistent with dynamical production from primordial binaries. Our investigation suggests that a triple to binary ratio up to  $\approx 2\%$  is consistent with a purely dynamical origin. If the observed number of triples is higher, then primordial triples have to be introduced in the theoretical modeling, as investigated for small  $N$  open clusters ( $N = 182$ ) by van den Berk et al. (2006).

## Acknowledgements

We thank the referee for a careful reading of the manuscript and John Fregeau for useful suggestions. MT was supported in part by NASA award HST-AR.10982.

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